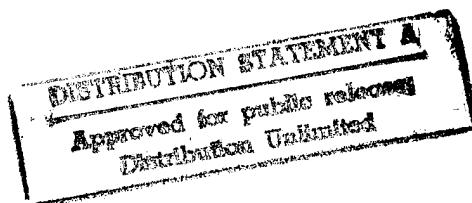


**Final Report on  
ONR GRANT-N00014-91-J-1700**

**Error Analysis in Numerical Solution  
of Fluid-Structure Interaction Problems**

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The scope of the work under the subject grant encompassed two interrelated subjects: deriving integral equation formulations for the fluid structure interaction problem and obtaining measures of the error incurred in the numerical solution of these equations. The ultimate goal is a reliable method of converting the mathematical model of how an elastic body reacts to excitation in a surrounding fluid into a form amenable to numerical solution and furthermore ascertaining useful bounds on the error inherent in the numerical approximation. Significant progress was achieved in pursuit of these goals.

With regard to the fluid-structure problem a comprehensive treatment of weak solutions of this problem was completed. A number of formulations were discussed and the delicate proofs of existence and uniqueness were obtained. This work is fundamental to proving convergence of numerical approximations such as Galerkin methods. Moreover it establishes the function spaces in which solutions should be sought, based on the physical principle of conservation of energy. The manuscript describing these results, item 4 in the attached list of papers prepared under the grant, has now been completed and will be submitted to the Quarterly Journal of Applied Mathematics and Mechanics.

The error analysis of numerical solution of integral equations, which was originally thought of only in the context of fluid-structure interaction, was generalized to a broad class of integral equations. The importance of meaningful a-posteriori error estimates was recognized as having a profound impact on all numerical solutions. Rather than extrapolating from accurate error measures which are available

only for very simple boundaries, the sphere for example where exact analytic solutions are known, to much more complicated boundaries where exact solutions are not known, work accomplished under this grant (items 1, 6, 13) provides the means for obtaining error measures for any shape. A systematic study of the different integral operators which occur in mathematical physics was begun. Results have been obtained for both first and second kind equations involving weakly singular and hypersingular integral operators. The appropriate function spaces in which to measure residual error were determined. It was shown how first kind equations may be well conditioned when considered in the appropriate Sobolev spaces. Moreover the computability of the norms in some of these spaces was demonstrated. Work remains to be done in pre- and post-processing to make the computations tractable in some cases and more efficient in all cases. Nevertheless the utility of a-posteriori error estimation has been established under this grant, and most importantly is being adopted by the boundary and finite element community. This, perhaps more than all of the papers and presentations prepared under the grant, a list of which follows, attests to the degree to which the goals of the grant have been achieved.

In addition to the work on fluid-structure interactions and error analysis in approximate solution of integral equations, a number of related problems in integral equations were treated. Significant results were obtained in coupling finite and boundary element methods, multigrid and other iterative methods, domain decomposition as well as some integral equation based results in inverse scattering. Papers describing these results are included in the following list.

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Papers Under ONR Grant-N00014-91-J-1700

1. M. Feistauer, G. C. Hsiao and R. E. Kleinman, Asymptotic and A-Posteriori Error Estimates for Boundary Element Solutions of Hypersingular Integral Equations, Center for the Mathematics of Waves, University of Delaware, Technical Report No. 91-18, to appear *SIAM J. Numer. Anal.*
2. M. Feistauer, G. C. Hsiao and R. E. Kleinman, Analysis of the Coupled Finite Element and Boundary Element Method for Exterior Nonlinear Pseudomonotone Problems, in preparation.
3. G. C. Hsiao and S. Zhang, Optimal Order Multigrid Methods for Solving Exterior Boundary Value Problems, *SIAM J. Numer. Anal.* **31**, 1994, 680-694.
4. G. C. Hsiao, R. E. Kleinman and G. F. Roach, Weak Solutions of Fluid-Solid Interaction Problems, in preparation.
5. R. E. Kleinman and P. M. van den Berg, A Modified Gradient Method for Two-Dimensional Problems in Tomography, *Journal of Computational and Applied Mathematics* **42**, 1992, 17-35.
6. G. C. Hsiao and R. E. Kleinman, Feasible Error Estimates in Boundary Element Methods, in *Boundary Element Technology VII*, C. A. Brebbia and M. S. Ingber, eds., Computational Mechanics Publications, Southampton, pp. 875-886, 1992.
7. R. E. Kleinman and P. M. van den Berg, Iterative Methods for Radio Wave Problems, *Review of Radio Science 1990-1992*, W. Ross Stone, ed., Oxford University Press, 1993, 57-74.
8. J. S. Asvestas and R. E. Kleinman, Electromagnetic Scattering by Indented Screens, Center for the Mathematics of Waves, *Trans. IEEE-AP*, **42**, 1994, 22-30.
9. G. C. Hsiao and W. L. Wendland, Domain Decomposition Via Boundary Element Methods, in *Numerical Methods in Engineering and Applied Sciences*, H. Alder, J.C. Heinrich, S. Lavanchy, E. Oate, and B. Suarez, eds. CIMNE, Barcelona, 1992, 198-207.
10. G. C. Hsiao and A. Cialdea, Regularization for some boundary integral equations of the first kind in mechanics, *Rendiconti dell'Accademia Nazionale dell Scienze detta dei XL*, to appear.
11. G. C. Hsiao and G. Gatica, *Boundary-Field Equation Methods for a Class of Nonlinear Problems*, Pitman Research Notes in Mathematics Series, 1995, 156 pp., submitted for review.

12. **G. C. Hsiao, M. Feistauer, R. E. Kleinman and R. Tezaur**, Analysis and numerical realization of coupled BEM and FEM for numerical exterior problems, in *Inverse Scattering and Potential Problems in Mathematical Physics*, R. E. Kleinman, R. Kress and E. Martensen, eds., pp. 47–73, Verlag Peter Lang, 1995.
13. **G. C. Hsiao and R. E. Kleinman**, Error analysis in numerical solution of acoustic integral equations, *Int. J. Numer. Meth. Eng.*, **37**, 1994, 2921–2933.
14. **P. Hähner and G. C. Hsiao**, Uniqueness theorems in inverse obstacle scattering of elastic waves, *Inverse Problems* **9**, 1993, 525–534.
15. **G. C. Hsiao**, On the progeny of Fichera's boundary integral methods, in *Problemi Attuali Dell'anaisi e Della Fisica Matematica*, P. E. Eicci, ed., Universita di Roma "La Sapienza", 1993, 126–144.
16. **R. E. Kleinman and B. Vainberg**, Full low-frequency asymptotic expansion for second order elliptic equations in two dimensions, *Math. Methods in the Appl. Soc.*, **17**, 1994, 989–1004.
17. **T. S. Angell, R. E. Kleinman and B. Vainberg**, Asymptotic approximation of optimal solutions of an acoustic radiation problem, in *Inverse Scattering and Potential Problems in Mathematical Physics*, R. E. Kleinman, R. Kress and E. Martensen, eds., Peter Lang, Frankfurt, 1995, 5–16.
18. **G. C. Hsiao**, On the boundary-field equation methods for fluid-structure interactions in *Problems and Methods in Mathematical Physics*, L. Jentsch and F. Trslitzsch eds., Teubner-Texte Zur Mathematik, Band 134, B. G. Teubner Verlagsgesellschaft, Stuttgart, 1994, 79–88.
19. **R. E. Kleinman and B. Vainberg**, Full low frequency asymptotic expansion for elliptic equations of second order in *Mathematical and Numerical Aspects of Wave Propagation*, R. E. Kleinman et al, eds., SIAM, Philadelphia, 296–301, 1993.

## **Presentations Under ONR Grant-N00014-91-J-1700**

1. **R. E. Kleinman and P. M. van den Berg**, Two-Dimensional Profile Reconstruction, Proc. of IEEE AP-S International Symposium, London, Ontario, Canada, June 24-28, 1991, pp. 1818-1821.
2. **R. E. Kleinman and P. M. van den Berg**, Profile Inversion for Two-Dimensional Scatterers, Proc. of the Progress in Electromagnetic Research Symposium PIERS, Cambridge, MA, July 1-5, 1991, p. 193.
3. **X. Jiang, R. E. Kleinman and P. M. van den Berg**, Iterative Methods for Electromagnetic Profile Inversion, Second International Conference on Industrial and Applied Mathematics ICIAM 91, Washington, DC July 8-12, 1991, p. 110.
4. **G. C. Hsiao**, Recent Advances in the Combined Methods of Boundary and Finite Elements, Int. Conf. on Industrial and Appl. Math. ICIAM 91, Washington, DC, July, 1991.
5. **G. C. Hsiao**, Domain Decomposition Via Boundary Element Methods, Special Session on Coupling of finite Element and Integral Equation Methods in Mechanics, First U.S. National Congress on Computation Mechanics, Chicago, IL, July 23-24, 1991.
6. **G. C. Hsiao**, Application of Boundary Element Methods to Problems of the Signorini Type in Elasticity, Special Session on Boundary Integral and Boundary Element Methods, First U.S. National Congress on Computational Mechanics, Chicago, IL, July 21-24, 1991.
7. **G. C. Hsiao**, Domain decomposition Via Boundary Element Methods, The Summer Conference on Domain Decomposition, Zentrum für Praktische Mathematik, Lambrecht, Germany, September 2-6, 1991.
8. **J. S. Asvestas and R. E. Kleinman**, The Far Field Scattered by Indented Screens, National Radio Science Meeting (URSI), Boulder, CO, January, 1992.
9. **G. C. Hsiao and R. E. Kleinman**, An Efficient Iterative Method for an Acoustic Boundary Integral Equation, AIAA 14th Aeroacoustics Conference Magnet Session Integral Formulation in Aeroacoustics and Aerodynamics, May, 1992, Aachen, Germany.
10. **R. E. Kleinman and P. M. van den Berg**, Iterative Methods for Intermediate Frequencies, IEEE/APS-URSI International Symposium, Chicago, IL, July, 1992.
11. **R. E. Kleinman and P. M. van den Berg**, An Extended Range Modified Gradient Technique for Profile Inversion, URSI International Symposium on Electromagnetic Theory, Sydney, Australia, August, 1992.

12. **R. E. Kleinman and G. F. Roach**, Obstacle Reconstruction From Backscattered Data, URSI International Symposium on Electromagnetic Theory, Sydney, Australia, August, 1992.
13. **G. C. Hsiao**, Boundary Integral Formulations for Plate Problems, SIAM 40<sup>th</sup> Annual Meeting, Los Angeles, CA,
14. **G. C. Hsiao**, Boundary-field equation methods for fluid-structure interaction, Euromech 316: Advanced Techniques in Structural Acoustics, University of Manchester, Manchester, England, April 11-14, 1994.
15. **G. C. Hsiao**, Boundary-field equation methods for fluid-structure interaction, Plenary lecture at "Numerical Modeling in Continuum Mechanics", Prag, Czech. Republic, August 22-25, 1994.
16. **G. C. Hsiao**, An integral equation method in inverse obstacle scattering of elastic waves, 6 Kolloquium des DFG-Schwerpunkts "Randelementmethoden", Schmallingenberg, Germany, September 27 - October 1, 1994.
17. **G. C. Hsiao**, A hybrid coupled finite-boundary element method for boundary-value problems in elasticity, Mathematisches Forschungsinstitut, Oberwolfach, Germany, October 2-8, 1994.
18. **I. V. Lindell and R. E. Kleinman**, Low frequency image theory for the dielectric sphere, National Radio Science Meeting, Boulder, CO., January 1993.
19. **G. F. Roach and R. E. Kleinman**, Modified Green's functions and obstacle reconstruction, British Applied Math. Colloq., Glasgow, Scotland, April 1993.
20. **R. E. Kleinman and P. M. van den Berg**, Modified gradient techniques for profile inversion, British Applied Math. Colloq., Glasgow, Scotland, April 1993.
21. **T. S. Angell, X. M. Jiang and R. E. Kleinman**, A new inversion technique for shape reconstruction, URSI Radio Science Meeting, Ann Arbor, MI, June 1993.
22. **R. E. Kleinman and P. M. van den Berg**, Profile inversion by simultaneous error reduction, XXIVth General Assembly of URSI, Kyoto, Japan, August 1993.
23. **G. C. Hsiao and R. E. Kleinman**, On the boundary-field equation methods for fluid structure interaction, Euromech 316, Advanced Techniques in Structural Acoustics, April 1994, Manchester, England.
24. **R. E. Kleinman**, New approaches to numerical solutions of integral equations, International Conference on Applied and Industrial Mathematics, June 1994, Linköping, Sweden.



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